Numerical Simulations of Fluid Flow and Heat Transfer in Pulse Tubes

A.E. Schroth[1], M. Sahimi[1], and C.S. Kirkconnell[2]

[1]University of Southern California, Los Angeles, California, USA
[2]Raytheon, El Segundo, California, USA

ABSTRACT

Pulse tube refrigerators give rise to many loss mechanisms, particularly within the pulse tube component, which greatly affect their overall thermodynamic efficiency. These losses are not well understood, as a comprehensive model that can take into account the effect of such loss mechanisms does not currently exist. In addition, the difficulty of obtaining accurate experimental data has contributed to this lack of understanding. In this paper, a comprehensive model is developed which simulates the flow process and heat transfer within the pulse tube component of orifice pulse tube refrigerators. In the model, the fluid oscillations are simulated using two dynamic meshes that move periodically and have a specified phase difference. The model includes the pulse tube and cold and warm end heat exchangers at the end of the pulse tube. Simulation results for the dynamic evolution of the temperature and velocity profiles along the tube axis are presented for several case studies. The simulations indicate the formation of a wall jet near the tube wall, in agreement with experimental observations. They also confirm the existence of momentum streaming, as well as radial heat conduction losses to the pulse tube wall. The thermodynamic performance of the tube is also quantified and compared with the available experimental data.

INTRODUCTION

Cryogenic temperatures are required for cooling devices such as infrared detectors, focal plane arrays, superconductivity devices and other scientific devices for atmospheric monitoring and astronomy. Specially designed cryogenic refrigeration systems, such as cryocoolers, were designed to meet the demanding requirements for space flight and long term on-orbit operation. Cryocoolers provide the necessary refrigeration such that these devices may be maintained at cryogenic temperatures for stable and effective operation. The most common types of cryocoolers used today, according to Radebaugh [2000], are the Joule-Thomson (JT), Brayton, Stirling, Pulse Tube (PT) and Gifford-McMahon (GM). In particular, Pulse Tubes have recently achieved Carnot efficiencies as high as about 20% at 80 K and temperatures as low as 2 K has been achieved in pulse tube refrigerators. This has propelled pulse tubes from a laboratory curiosity to the point where it is now considered the standard for many space applications.

There are two theories that describe the refrigeration mechanism in the basic pulse tube refrigerator. In the surface heat pumping theory [Gifford and Longsworth, 1966], this phenomena was explained based on a thermodynamic analysis of transient heat transfer between

Cryocoolers 13, edited by R. G. Ross, Jr.
Springer Science+Business Media, Inc., New York, 2004

the fluid and its surrounding structure. At low frequency, surface heat pumping dominates, an effect that diminishes as the operating frequency increases. The pulse tube refrigeration mechanism was reanalyzed on the basis of enthalpy flow [Radebaugh, 2000] for an orifice pulse tube refrigerator and attributed to the cycle-averaged enthalpy flux in the pulse tube as a result of the phase shift effect between the mass flow rate and the pressure fluctuations. The theory behind PT refrigerators is very similar to that of the Stirling Refrigerators, except for the added benefit of having no moving parts in the cold end.

Pulse Tube Loss Mechanisms

The movement of the piston creates pressure oscillations throughout the PT refrigerator resulting in an oscillatory effect on the gas flow. The dynamic nature of this effect, in particular within the pulse tube component, and how this contributes to PT losses, is the focus of this research. The known secondary flows [Kirkconnell, 1995] that arise due to the interaction of the core and boundary layer flow necessitate a 2^{nd} order model. To date, numerical modelling of flows within the pulse tube component has been very limited and thus the loss mechanisms are not clearly understood, much less quantified. The losses occur when there are changes in enthalpy flow resulting in fluctuations in amplitude of the temperature oscillations. This is an irreversible process that generates entropy. Thus, this process is a loss mechanism [Lee et al., 1994]. In pulse tubes, these losses occur at the transition between heat exchangers (isothermal regions) and adiabatic regions.

The gross refrigeration power produced by a pulse tube cryocooler is further reduced by losses occurring in both the regenerator and in the pulse tube. Losses within the regenerator have been calculated accurately using various software packages, such as REGEN (developed at National Institute of Standards and Technology), DeltaE (Los Alamos National Laboratory), and ARCOPTR (Ames Research Center Orifice Pulse Tube Refrigerator). REGEN is a model for the regenerator only, incorporating the time-dependence of the oscillating parameters. It handles large amplitude oscillations as long as the ensuing pressure drops through the regenerator are small [Radebaugh et al., 1994]. ARCOPTR is based on one-dimensional thermodynamic equations for the regenerator and assumes all mass flows, pressure and temperature oscillations are small and sinusoidal. The results from ARCOPTR are the oscillating pressures, mass flows and enthalpy flows in each of the main components of the cooler [Roach and Kashani, 1996]. In comparison, DeltaE solves the one-dimensional wave equation based on the acoustic approximation (small amplitude). This software does not include any nonlinear effects resulting from high amplitudes. It is a linearized model that treats only the lowest order sinusoidal component of the oscillations. It can treat regenerators with large pressure drops and it used correlations for heat flow and friction that include non-linear effects that occur at high velocities [Swift and Ward, 1994].

The losses occurring within the pulse tube can also be calculated. However, the underlying cause of these losses is still not well understood and far from being quantified. The characterization of flows within pulse tubes cryocoolers has been a subject of both experimental and analytical interests in recent years. And though more efficient pulse tube configurations have been developed, the refrigeration losses in the pulse tube component have remained a significant percentage of the total available gross refrigeration capacity of these devices.

Some of these pulse tube refrigeration losses can be attributed to convective heat transfer within the pulse tube [Lee et al., 1994], which carries heat from the hot heat exchanger to the cold heat exchanger and thereby reduces the net cooling power. This convective driving occurs in the oscillatory boundary layer at the solid wall of the pulse tube. In the cyclic expansion and compression process, the gas elements close to the wall experience different viscous drag by the alternation of gas temperature and experience a net drift from the cold end to the hot end (called secondary mass flux). The Reynolds stresses in the viscous layer produce the forces that drive steady momentum streaming [Stuart, 1963]. The steady secondary mass flux is a quadratic quantity and thus nonvanishing when time-averaged over a cycle. According to [Lee et al., 1994], the steady mass flow is strongly dependent on the velocity amplitude, velocity gradient,

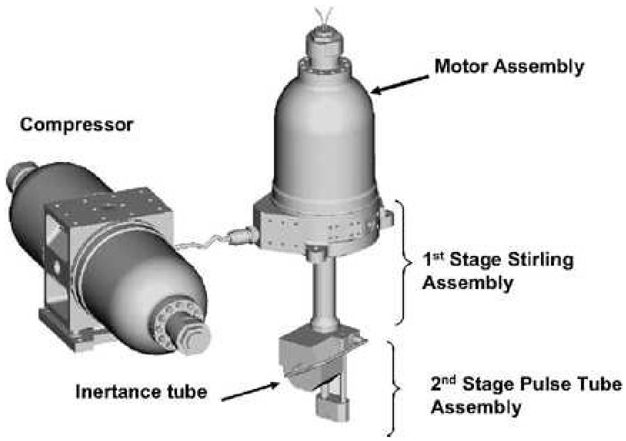

Figure 1. RSP2 Cryocooler cold head design

and on the velocity phase angle between the tube ends. Since the mass flux along the pulse tube must be zero, a flow streaming phenomena occurs in the center portion of the pulse tube, compensating for the secondary mass flux.

In orifice pulse tubes, an appropriate phase angle between velocity and pressure is required to produce favorable enthalpy flow. In a 2D linearized model developed by Lee et al. [1994], they concluded that the heat transfer between the gas and the tube wall produces enthalpy flow which is rejected to the tube wall and not to the hot heat exchanger. The necessity of a 2D model to understand and quantify these nonlinear effects was also demonstrated by Schroth et al. [2002]. Jeong [1996] investigated the same secondary flows in basic pulse tubes and confirmed the existence of large-scale streaming and the effects of axial temperature gradient. He showed analytically that the magnitude of the secondary flow increases as the axial temperature gradient (difference between the cold and hot heat exchangers of a pulse tube) increases. The entropy flow is negative at low temperatures (cryogenic), but positive at higher temperatures was reported by Radebaugh. [2003].

In the present study, a numerical model for simulating the detailed dynamic performance and characteristics of the oscillating flow in the pulse tube refrigerator is developed using FLUENT CFD software. The model simulates only the 2nd stage portion of the Raytheon Stirling/Pulse Tube Two Stage (RSP2) cryocooler [Finch et al., 2003] (see Figure 1). Numerical simulations are performed to study the various pulse tube loss mechanisms and their effect on overall performance and efficiency. Predictions made from the simulation are compared with the experimental data obtained from the RSP2 cryocooler in which good agreement was found for various operational configurations. Case studies are then developed in which investigations are made to determine how thermodynamic performance is affected by changes made to the properties of cold and warm heat exchangers. These changes include screen mesh size as well as overall heat exchanger lengths.

PHYSICAL AND SIMULATION MODEL

RSP2 Cryocooler

As previously mentioned, Raytheon is developing a two-stage Stirling/pulse tube cryocooler (RSP2) for long life space infrared (IR) sensor applications. This cryocooler, whose first stage utilizes a conventional Oxford-class Stirling expander, contains a second state pulse tube, in a U-turn configuration, whose expander is in intimate contact with the first stage cold end. This research contained in the paper focuses primarily on the simulation only the 2nd stage portion of the RSP2 cryocooler.

The Raytheon RSP2 cryocooler contains a unique feature in that it can vary refrigeration at two locations, having different operational temperatures, while still maintaining a relative

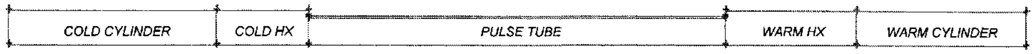

Figure 2. 2D Simulation Model (2nd Stage only, symmetric along bottom center-line)

constant efficiency. In other words, the refrigeration capacity, at each location, can be shifted, in real time, by adjusting the phase angle between the compressor and the expander piston, with minimal change in system efficiency. This unique advantage allows the cooler to perform over a wider range of load requirements compared to other approaches.

The RSP2 cryocooler was selected for simulation since a large performance database has been collected during its development and a correlated system level model has been developed. The experiment data contained within this database includes results obtained while varying the operating conditions and temperatures. This database forms the foundation of the model correlation efforts that will be conducted in this study. One of the key objectives of this research project is to perform numerical studies such that comparisons can be made between computational fluid dynamic (CFD) predictions to that of experimental data available from the RSP2 performance database.

Besides the regenerator, the 2nd stage portion of the RSP2 cryocooler consists of a cold and warm copper heat exchanger, and a pulse tube. The porosity of the heat exchangers is nominally from 0.60 to 0.75 depending on the size of the wire screen mesh. The operating wall temperatures of the cold and hot end heat exchangers, for the cases to be evaluated, varies from 45 K and 110 K. The operating mean pressure is nominally 3.0 MPa, with operational frequencies ranging from 34 Hz to 45 Hz.

Simulation Model

The RSP2 simulation model, shown in Figure 2, is representative of a Stirling/Pulse tube second stage assembly whose sizing is based upon test results from the RSP2 development program. The purpose of developing this model was to be able to predict the performance of the RSP2 Hybrid cryocooler based on the available experimental data. This would be a direct 2D simulation of the fluid flow and heat transfer occurring within the second stage portion of the cryocooler.

The key to this model was the development of a simulation approach that would produce the similar characteristic phase angle between the pressure and velocity waveforms, between the cold and warm sides of the pulse tube, as observed from the data collected from the RSP2 experimental database. These phase angles were produced by modeling a piston cylinder arrangement on both ends of the simulation model, whose piston surfaces form the outer edges of the grid network while still maintaining a closed system and conserving mass.

With the grid network meshed (i.e., cold and warm heat exchangers and pulse tube), the outer surfaces were position controlled, to following a period motion, varying the total volume within its network. Such movements would dynamically deform the internally meshed cells, removing and adding cell over its period. These surfaces followed a periodic motion, but with a controlled phase angle between their respective motion. The moving surfaces of the grid network, which deform the interior meshed cells, act as virtual *moving pistons*, oscillating the flow while maintaining conservation of mass. Extensive research was involved in the development of custom algorithms that automatically control the position of each moving surface, relative to each other. It was important that the simulation of the moving surfaces, for a closed system, would adequately conserve mass and energy. In many cases, it was found that the artificial movement of surfaces, if done improperly, could produce an oscillating change in total mass of the system, over a cycle, that could effect the overall system energy balance and produce instability in the numerical solution.

The simulation model consists of two pistons, cold and warm heat exchanger, and the pulse tube. The solver uses a finite volume approach to solve the governing equations describing the heat transfer and fluid flow in this system. Taking advantage of the symmetrical aspects of this

Table 1. Properties of copper wire screens

Mesh Size	Wire Diameter (mm)	Porosity	Mesh Distance (mm)	Hydraulic diameter (mm)
150	0.061	0.6993	0.140	0.1418
250	0.041	0.6582	0.120	0.0788
300	0.031	0.6938	0.100	0.0704
400	0.025	0.6677	0.083	0.0504

problem, only a 2D representation of the RSP2 Hybrid cryocooler was modeled. The symmetry axis (lying on the x-coordinate axis) runs down the centerline of the pulse tube. For 2D problems, the solver computes all integral quantities for an angle of 2π. Each area of interest was designated a separate face zone. A 2D regular, structured grid of mesh elements was then generated for each face zone using quadrilateral mesh elements. Use of the quadrilateral mesh element scheme is applicable primarily to faces that are bounded by four or more edges. As previously mentioned, the numerical model only simulates the 2nd stage portion of the RSP2 Hybrid cryocooler consisting of cold and warm heat exchangers, pulse tube, and pulse tube wall.

The pulse tube wall was modeled having material property of steel. The heat exchangers were modeled as homogeneous porous zones consisting of copper material, having a pore structure that exhibited a viscous and inertial resistance corresponding to that of mesh stacked copper interwoven meshed screens. Various sizes of mesh screens were evaluated as part of this study. Porosity values ranging from 0.60 of 0.75 were evaluated for each of the heat exchangers. The cold and warm pistons, piston cylinder walls, and heat exchangers were modeled as isothermal surfaces contributing to the overall heat transfer. The working fluid was modeled as ideal helium gas. Compressibility effects were assumed negligible based on the temperature range in which the available data was collected (45K to 120K).

Based on experimental mass flow rates, a laminar flow velocity formulation was selected for the solver. The predicted velocities were found to be well within the laminar flow range so the use of a turbulent model formulation was not required. The 2D axissymetric model assumes that no swirling flows within the 2nd stage portion of the cryocooler. There is no evidence from the experimental data that would suggest the existence of any significant amount of swirling flow.

Case Studies Definition

An area of interest regarding pulse tube cryocoolers is the effect of flow straightening caused by the cold and warm heat exchangers and its subsequent effect on thermodynamic performance. Flow straightening can occur by modifying the length of the heat exchanger as well as by changing the flow matrix properties (i.e., mesh size). The heat exchangers, in the RSP2 cryocooler consist of stacked copper meshed screens. The effect of changing the mesh size inside the heat exchanger should produce changes in pressure drop as well as the phase angle between the pressure and velocity wave. Variations in mesh size, regarding regenerators , has been reported by both Zhao et al. [1998] and Harvey [1999] in which similar conclusions have been drawn on the effect on pressure drop and resultant change in phase angle between the velocity and pressure wave.

Among many research groups, the design length of cold and warm heat exchangers, within pulse tube cryocoolers is not an exact science. There exist many rule-of-thumb standards for determining an adequate heat exchanger design length, but whose methods are rarely discussed, much less published. Often, the methods are regarded as proprietary information. In any event, the goal of this study is to shed some light on the effects on changes in heat exchanger lengths using predictions from the 2D simulation.

As mentioned earlier, changes in these flow characteristics are also dependent on the size of screen mesh used within the heat exchangers. The screen mesh sizes that will be evaluated, using the simulation model, includes 150, 250, 300, and 400 mesh screens made from interwoven copper strands. Since the simulation model is a 2D representation of the heat exchanger, the effective properties of the screen mesh (i.e., porosity, inertial and viscous

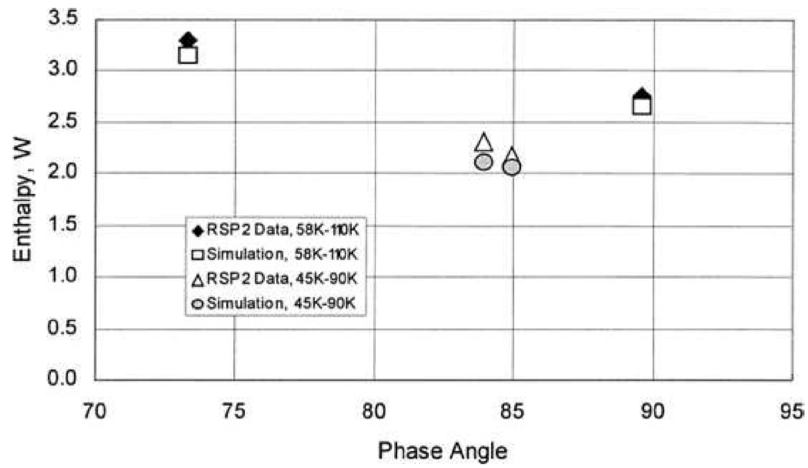

Figure 3. Comparison of enthalpy flow rates in pulse tube between experimental and predicted values for separate operational conditions.

resistances) are used in the formulation of the model. Table 1 describes the various screen mesh sizes evaluated and their properties.

SIMULATION RESULTS AND MODEL VALIDATION

Verification of the simulation model was first carried out by first establishing an internal energy balance and thus verifying the amount of work input into the system balanced by the total heat transfer occurring at the various surfaces. After which, predictions made from the model, was compared to the RSP2 cryocooler experimental data for various operating conditions.

Model Verification

In these simulations, the amount of PV compression and expansion power generated by the warm and cold pistons, respectively, was compared to the total surface heat flux, integrated over each of the isothermal surfaces. The PV power was calculated at each time step based on the product between the average absolute pressure and the rate of change in total volume. These values were then integrated over the cycle using Simpson's rule and divided by the characteristic time τ. The total heat transfer on the various surfaces were calculated using the same approach (i.e., cycle-averaged). Equation 1 shows the calculation used to determine the PV power:

$$PV\ \text{Power} = \frac{1}{\tau}\int_0^{\tau} p\frac{dV}{dt}d\tau, \qquad (1)$$

where p is the absolute pressure, V is the volume, and τ is time. The rate of change of volume was based on the difference between the volume at the current times step and the previous time step divided by the time step size. The energy balance performed on each of the simulations indicated an error of less than 5% of the total energy of the system. In other words, the PV power compression plus the net heat transfer on the cold side balanced that with the PV power of expansion and net heat transfer on the warm side. Most of the error can be attributed to a poorly meshed grid near the isothermal surfaces. Future models will be performed using a higher grid mesh resolutions.

In comparing the results from the 2D simulation model to the RSP2 data, it was important to focus on the enthalpy flow in the pulse tube, since this represents the available gross refrigeration. Enthalpy flow in the pulse tube component was calculated using Equation 2 and then cycled averaged. These predictions, for various flow configurations were compared to the actual enthalpy flow as demonstrated from the RSP2 development database.

$$h_{PT} = \rho U_{axial} c_P T \,, \tag{2}$$

The results are shown in Figure 3 in which the enthalpy in the pulse tube is calculated for a particular phase angle at two operation temperatures. These comparisons made in this figure were made at two separate operational temperatures settings and at various phase angles. The results show very good agreement the experiment data. Based on the preliminary results, the simulation appears to underestimate the enthalpy flow through the pulse tube in both cases that were examined. The total error between the actual and simulation was calculated to be less than 10% overall.

Figures displaying the pressure versus overall volumetric change of the working gas are shown in Figure 4. The PV diagram (bottom) shows the refrigeration performance of one example simulation model configuration.

Case Studies Predictions

Having completed the energy balance, the model was then exercised for a set of experimental conditions corresponding to the RSP2 cryocooler experimental data points. The results obtained show a definite trend in cryocooler performance as the entrance effects/mixing is reduced at the ends of the pulse tube. This coarseness of this current preliminary model prohibits the direct assessment of meaningful flow patterns. However, changes in heat exchanger pressure drop, as well as the changes in phase angle (between the pressure and velocity wave), is shown

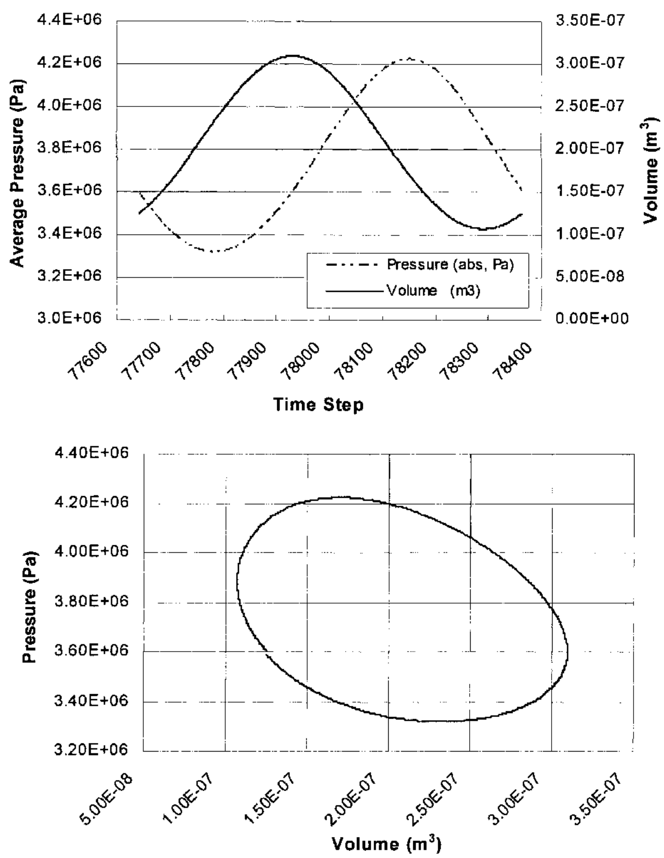

Figure 4. PV Diagrams resulting from the simulation model (cold side)

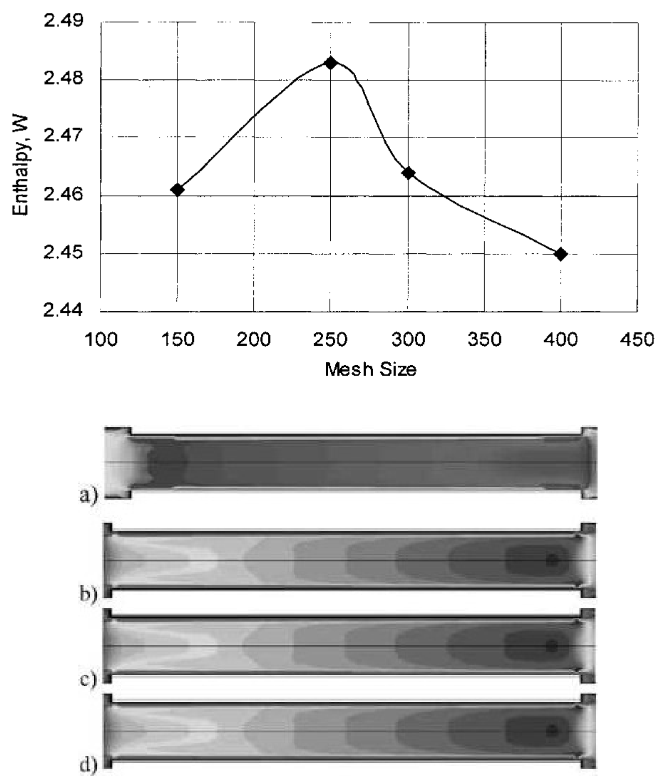

Figure 5. Variation in Enthalpy (gross cooling power) across the pulse tube as a function of mesh size. Below, velocity contours for the various mesh sizes (symmetry axis mirrored at center-line) 5a) 400, 5b) 300, 5c) 250, 5d) 150

to affect the overall gross cooling power flowing through the pulse tube (see Figure 5, top). The reduction in enthalpy flow, when the mesh screen sized is increased, was observed to be small but not negligible. Figure 5a-d illustrates the influence of the mesh size on flow straightening in regards to the velocity profiles. For each sub-figure shown, a contour plot of the velocity is shown, corresponding to the various sized screen mesh, at the exact same crank angle position. Notice the decrease amount of disturbance caused by the increased size of the mesh screen size. As the mesh size decreases, the effect on flow straighten becomes less and less effective as expected. The changes between the 150 and 250 wire mesh size are practically indistinguishable. For this preliminary model, the coarseness of the grid mesh applied to this simulation, does not allow for an immediate qualitative analysis of the resultant flow structure. However, immediate work is currently in progress to develop adequately refined meshes to ascertain these effects.

As the mesh size is increased from 150 through 400 mesh screens, it was observed that the pressure drop across the cold heat exchanger increased as expected, though very small. These resultant changes in the pressure drop was also accompanied by changes in phase angle between the pressure and velocity waves, which was also expected.

Figure 6 shows the effect of varying the length of the cold and warm heat exchanger lengths and its effect on the flows within the pulse tube. Based on this preliminary model, it appears that changes in overall heat exchanger length have a negligible effect on performance (i.e., enthalpy flow). It is apparent that the current model requires a much finer grid to resolve these effects. The flow fields, shown in Figure 6, show very little difference for each heat exchanger length.

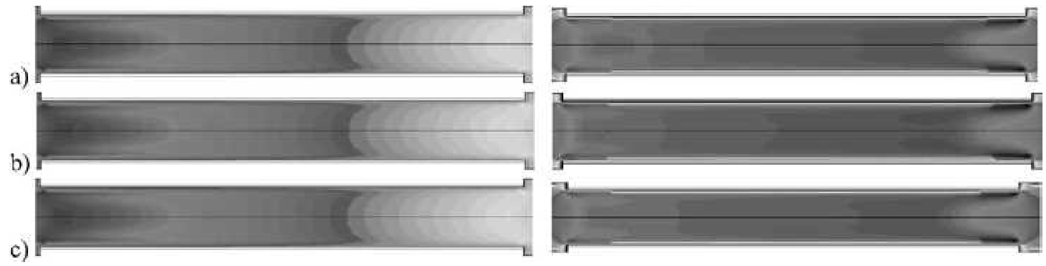

Figure 6. Velocity contours for various heat exchanger lengths corresponding to percentage increases from the reference size (symmetry axis mirrored at center-line) a) 0%, b) -10%, c) -20%

RESULTS AND CONCLUSIONS

A 2D model of the fluid flow and heat transfer, modeling the 2nd stage portion of a Stirling/Pulse Tube cryocooler was presented which was correlated to Raytheon's RSP2 cryocooler development work. Model validation efforts produced excellent agreement based on energy conservation and prediction of enthalpy flow across the 2nd stage pulse tube component.

A prediction on the effect of changes in wire mesh size was performed using the simulation model. The changes in mesh screen size, within the cold and warm heat exchanges, affected both the pressure drop and phase angle relationship of the oscillating flow. As the mesh size increased, both the pressure drop and phase angle increased. The current preliminary model could not resolve differences in thermodynamic performance when heat exchanger lengths were increased. Further action to increase model resolution is currently progress.

ACKNOWLEDGMENT

This work was supported by Raytheon Company under a grant given to the University of Southern California.

REFERENCES

1. Finch, A.T., Price, K.D., and Kirkconnell, C.S., "Raytheon Stirling/Pulse Tube Two-Stage (RSP2) Cryocooler Advancements," *Adv. in Cryogenic Engineering*, Vol. 49B, Amer. Institute of Physics, Melville, NY (2004), pp. 1285-1292.
2. Gifford, W.E. and Longsworth, R.C., "Surface Heat Pumping", *Advances in Cryogenic Engineering*, vol. 11 (1966), pp. 171-179.
3. Harvey, J.P., Master's Thesis, Georgia Institute of Technology (1999).
4. Jeong, E.S., "Secondary Flow in Basic Pulse Tube Refrigerators," *Cryogenics*, vol. 36 (1996), pp. 317-323.
5. Kirkconnell, C.S., *Numerical Analysis of the mass flow and thermal behavior in high frequency pulse tubes*, PhD Thesis, Georgia Institute of Technology (1995).
6. Lee, J.M., Kittel, P., Timmerhaus, K.D., and Radebaugh, R., "Steady secondary momentum and enthalpy streaming in the pulse tube refrigerator," *Cryocoolers 8*, Plenum Publishers, NY (1995), pp. 359-369.
7. Radebaugh, R., "A review of pulse tube refrigeration," *Adv. in Cryogenic Engineering*, vol. 35 (1990), pp. 1191-1206.
8. Radebaugh, R., "Development of the pulse tube refrigerator as an efficient and reliable cryocooler," *Proceedings of the Institute of Refrigeration of London (2000).*
9. Radebaugh, R., "Low Temperature and Cryogenic Refrigeration," *Series II: Mathematic, Physics and Chemistry*, vol. 99, Kluwer Academic Publishers, New York (2003), pp. 415-434.

10. Radebaugh, R., O'Gallagher, and A., Gary, J., *A numerical model for regenerator performance*, NIST (1994).

11. Roach, P.R., and Kashani, A., "A Simple modeling program for orifice pulse tube coolers," *Cryocoolers 9*, Plenum Press, NY (1997), pp. 327-334.

12. Schroth, A.E., Kirkconnell, C.S. Kirkconnell, Sahimi, M., "Numerical Model for Pulse Tube Using Method of Lines," *Cryocoolers 12*, KluwerAcademic/Plenum Publishers, New York (2002), pp. 379-387.

13. Stuart, J.T., *Laminar Boundary Layers*, Oxford University Press (1963), Chapter 7.

14. Swift, G.W., and Ward, W.C., "Design environment for low amplitude thermoacoustic engines," *Journal of the Acoustical Society of America*, vol. 95 (1994), p. 3671.

15. Zhou, Y., Ju, Y., and Jiang, Y., "Experimental Study of Oscillating flow characteristics for a regenerator in a pulse tube cryocooler," *Cryogenics*, vol. 38 (1998), pp. 649-656.