COMPARISON OF ENTROPY GENERATION RATES IN VARIOUS MULTI-STAGE STIRLING-CLASS CRYOCOOLER CONFIGURATIONS

C. S. Kirkconnell[1], J. P. Harvey[1], and P.V. Desai[2]

[1]Raytheon Company
El Segundo, California, 90245, USA

[2]Georgia Institute of Technology
Atlanta, Georgia, 30332, USA

ABSTRACT

Internal irreversibility, hence entropy generation, occurs in cryocoolers due to frictional effects and heat transfer across finite temperature gradients. As these effects are finite in any real system with relative motion between a flowing fluid and the system boundaries, entropy is generated within every component in a cryocooler. The challenge for the designer is to determine, and ultimately minimize, the total rate of entropy generation for the cryocooler, i.e., the sum of all the component entropy generation rates. Of present interest are the comparative entropy generation rates of various Stirling-class configurations for multistage cryocoolers. This includes Stirling moving displacer systems, pulse tube systems, and "hybrid" combinations of Stirling and pulse tube technologies. As is demonstrated herein, the rate of entropy generation is strongly dependent on the thermodynamic configuration of the refrigerator. Entropy generation rates in each component are affected by configuration because configuration in large part drives the thermodynamic operating conditions, in particular mass flow rate and pressure-to-mass flow phase angles, throughout the cryocooler. Thus regenerator friction and heat transfer losses, which tend to dominate the total rate of entropy generation in a cryocooler, are directly impacted by the method of cryocooler staging. The quantitative results of the comparison are presented together with a case study on how entropy generation can be reduced in "hybrid" systems through load shifting.

INTRODUCTION

The majority of space cryogenic cooling needs today are best met with Stirling-class cryocoolers. (The term "Stirling-class" herein includes pulse tube cryocoolers with no moving parts in the expander as well as traditional Stirling cryocoolers with a moving

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displacer piston.) Applications of present interest predominantly involve cooling infrared sensor focal plane arrays and sometimes the associated optics. In the case where both the focal plane array (FPA) and optics must be cooled to cryogenic but different temperatures, the most thermodynamically and mass efficient solution is provided through the use of a multistage cryocooler. Similarly, space applications involving the cryogenic cooling of superconducting circuits and warmer, but still cryogenic low noise amplifiers are in general most efficiently satisfied with a multistage cryocooler. This study is a continuation of an ongoing effort by the authors to determine how Stirling-class technology is best applied to meet these and other multistage cryogenic application needs.

Multistage combinations of Stirling-class technology include multistage pulse tubes [1, 2], multistage Stirlings [3], "hybrid" combinations of pulse tube and Stirling [4]. For the purpose of the present effort, the scope of the investigation has been limited to the consideration of two-stage cryocoolers, which is of the greatest practical interest given the current state of industry-wide development efforts and the likelihood of near-term availability of two-stage cryocoolers for space applications. The specific configurations of interest are described herein.

A numerical study has been conducted in which the competing configurations have been modeled using ground rules intended to normalize the results to enable meaningful comparison. The goals of the study were to determine the relative thermodynamic efficiency potential of the various configurations and to identify the dominant loss mechanisms in each. This was accomplished through a series of system-level optimizations followed by component-level evaluation of the contributing sources of entropy generation. The results provide insight into the relative thermodynamic strengths and weaknesses of the competing design approaches.

As discussed herein, hybrid systems provide the capability to shift load capacity between stages through control of the Stirling displacer motion phase angle relative to the pressure wave. A case study has been performed to determine the sensitivity of efficiency on mid-stage temperature for a particular Stirling/pulse tube hybrid cryocooler. A roll off in efficiency away from the design point is demonstrated. Therefore, a follow on study was performed to evaluate the utility of load shifting to improve efficiency over a broad range of mid-stage temperatures. Results are provided herein.

TWO-STAGE CRYOCOOLER CONFIGURATIONS

Four multi-stage combinations of Stirling and pulse tube technology were considered: two-stage Stirling, two-stage pulse tube, first-stage Stirling / second-stage pulse tube (Stirling / pulse tube), and first-stage pulse tube / second stage Stirling (pulse tube / Stirling). Beyond these basic definitions, there are many permutations that relate to physical construction. For example, multistage Stirling coolers can be constructed using either a single two-stage piston or two single-stage pistons; cryocoolers with pulse tube second stages can use either an ambient or cryogenic surge volume. An assumed physical construction of each of the configurations was made based upon the most common approaches taken by space cryocoolers manufactures for the two-stage Stirling, two-stage pulse tube, and Stirling/pulse tube hybrid [FIGS 1-3]. The authors are unaware of any attempts to construct a pulse tube/Stirling. A physical configuration was assumed that mirrors the other thermodynamic designs in that the second stage warm end is thermally sunk to the first stage cold end [FIG 4]. The mechanical challenges of driving an all-cryogenic piston are recognized, but the objective of comparing the performance of similarly-constructed thermodynamic designs was satisfied by this selection.

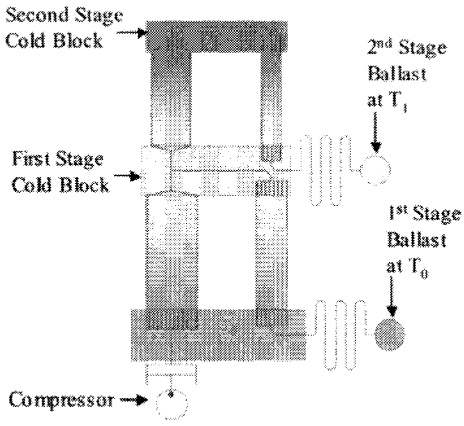

FIGURE 1. Two-Stage Pulse Tube.

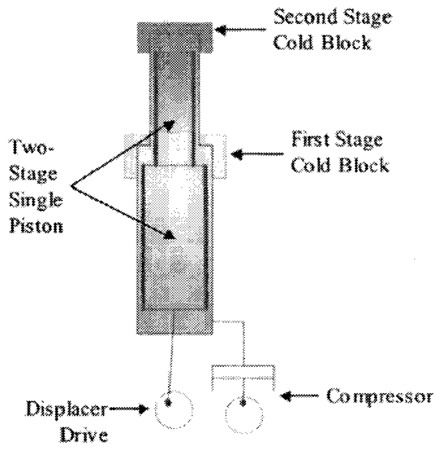

FIGURE 2. Two-Stage Stirling.

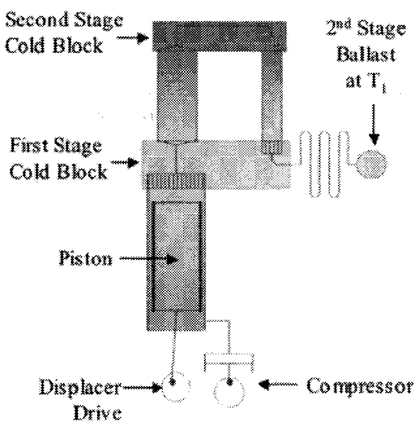

FIGURE 3. Stirling / Pulse Tube Hybrid.

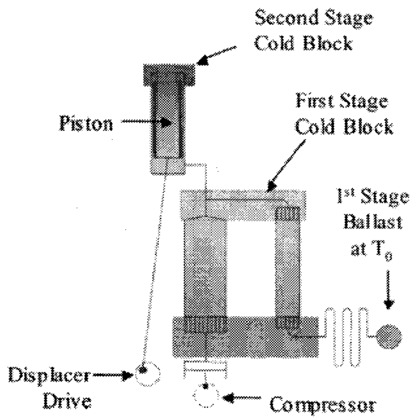

FIGURE 4. Pulse Tube / Stirling Hybrid.

NUMERICAL EXPERIMENTS

To create a normalized basis from which to compare the competing configurations, some baseline parameters were established that are common to all cases. The most basic of these are:

- Operating frequency = 45 Hz;
- Mean pressure = 3.4 MPa;
- Input pressure-volume (PV) power from the compressor = 40 W.

The first two are within the typical range of frequency and mean pressure for modern space cryocoolers, but the 40 W input power is on the low side for most applications. This power level was selected for similarity to the existing Raytheon / Georgia Tech pulse tube cryocooler so that results from the pulse tube code could be directly compared to experimental data. Similar comparisons of the Stirling code to the Raytheon Stirling Single-Stage (RS1) cryocooler and the Stirling / pulse tube code to the Raytheon Stirling / Pulse Tube Two-Stage (RSP2) cryocooler provided confidence that the software models are sufficiently accurate to yield valid comparisons.

One of the chief objectives of the study was to determine if the optimum thermodynamic configuration is a function of operating temperature. The configurations described in Figure 1-4 were therefore exercised at three distinct temperature levels:

- 35 K first stage, 80 K second stage;
- 60 K first stage, 100 K second stage;
- 100 K first stage, 150 K second stage.

Each configuration was optimized at the 60 K/100 K, and the design geometry and operating conditions were held constant for characterization at the high and low temperature extremes. This represents the usual case where in practice a cryocooler has been designed to operate at some temperature and refrigeration load other than that for which a specific payload requires cryogenic cooling.

To facilitate comparative evaluation, the performance of each cryocooler was represented with a single figure of merit, the "effective net refrigeration at the second stage," defined and described by Kirkconnell and Price [5]:

$$\dot{Q}' = \dot{Q}_{1,net}\frac{\beta_{2,C}}{\beta_{1,C}} + \dot{Q}_{2,net} \tag{1}$$

where the Carnot coefficient of performance (COP) for the i[th] stage is defined in terms of the cold block and ambient (T_0) temperatures as follows:

$$\beta_{i,C} = \frac{T_i}{T_0 - T_i}. \tag{2}$$

The rate of entropy generation is minimized when the Second Law COP, β_{II}, is maximized:

$$\beta_{II} = \frac{\dot{W}_C}{\dot{W}_{act}}. \tag{3}$$

where $\dot{W}_{act}$ is the actual total input power and $\dot{W}_C$, the Carnot power, is defined by

$$\dot{W}_C = \frac{\dot{Q}_1}{\beta_{1,C}} + \frac{\dot{Q}_2}{\beta_{2,C}}. \tag{4}$$

Both $\dot{Q}'$ and β_{II} were used to rank and characterize the configurations as discussed in the next section.

MODELING RESULTS

FIGURE 5 is a graph of $\dot{Q}'$ versus second stage temperature at constant input power for the four configurations studied. The two-stage Stirling yields the highest refrigeration rates over the range considered, followed by the Stirling / pulse tube. The performance of these two is almost identical at 100 K, while the Stirling / pulse tube performance drops off relative to the two-stage Stirling with decreasing temperature, though this is somewhat misleading given that the two-stage Stirling load split is heavily tilted towards to the first stage for the low temperature case, as is subsequently discussed. The two-stage pulse tube is the poorest thermodynamic performer over the entire range. The pulse tube / Stirling trends with the two-stage pulse tube with only slightly better performance.

These results agree with intuition. The two-stage Stirling requires the smallest total working volume, lowest flow rates, and provides partial recovery of the cold end gross refrigeration as mechanical energy at the warm end of the displacer piston. The Stirling / pulse tube hybrid, as has been previously discussed [6] is efficient in part because it purposefully locates the characteristically high flow rate pulse tube portion at the second stage where high density begets low velocities and hence low friction losses. The pulse tube / Stirling configuration results in high first-stage flow rates and resulting high first-stage friction losses. This characteristic becomes less detrimental as the average velocities in the cooler decrease with decreasing temperature; note the near convergence with Stirling / pulse tube results at 35 K. The two-stage pulse tube, while having the desirable characteristic of being the mechanically simplest, is the inherently least inefficient because the gross refrigeration is dissipated as frictional heat in the inertance tubes at both stages.

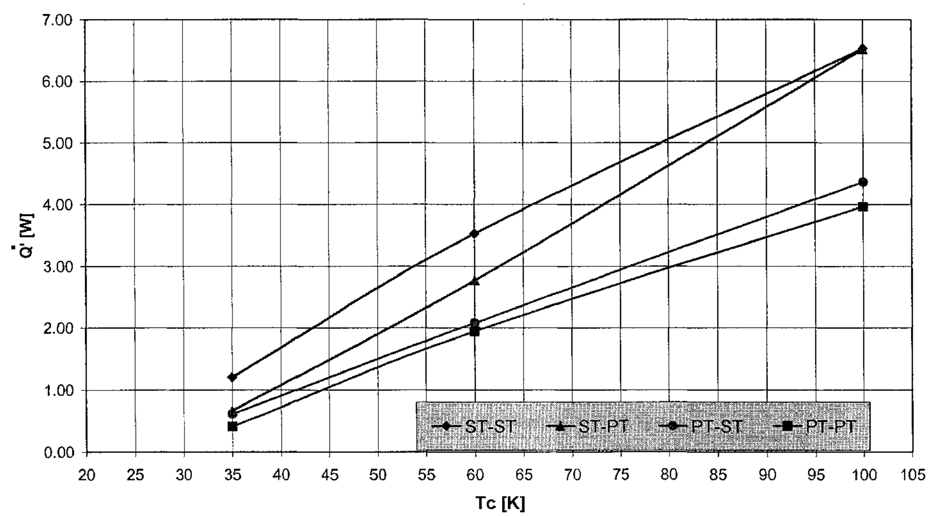

FIGURE 5. Normalized Second Stage Refrigeration Capacity for Two-Stage Cooler Configurations. ST-ST = two-stage Stirling; ST-PT = Stirling / pulse tube hybrid; PT-ST = pulse tube / Stirling hybrid; PT-PT = two-stage pulse tube. Tc = coldest stage temperature (i.e., second stage temperature).

A comparison of β_{II} versus temperature is more illustrative in that it reveals not only the relative performance of the four configurations, but it also shows how the entropy generation rate trends with temperature. Of particular note is that β_{II} increases with temperature for the Stirling / pulse tube over the entire range despite the fact that it, like the others, was optimized at the mid-point of the temperature range. The other three configurations have peak efficiencies near the mid-point. This tends to indicate that the Stirling / pulse tube is increasingly efficient relative to the other configurations as temperature increases.

For the baseline 60 K / 110 K operation at which each configuration was optimized, one of the constraints was a first stage-to-second stage net refrigeration ratio of 3:1 as typical of common applications. The 3:1 constraint was retained for the hybrid designs at the high- and low-temperatures because the hybrids possess the ability to shift refrigeration between the stages at constant input power simply by changing the Stirling piston phase angle relative to the pressure wave. (Recall the previously defined constraint of constant 40 W input power for all cases.) The 3:1 load ratio constraint was relaxed out of necessity for the other configurations; the two-stage pulse tube has no active phase shift mechanism, and the two-stage Stirling first- and second-stages are essentially phase locked with the single piston design. Though it was attempted, it proved impossible to simultaneously maintain both a 3:1 load ratio <u>and</u> constant input power for the two-stage Stirling.

The importance of this direct load shift capability is evident from consideration of the low-temperature results. The Stirling / pulse tube and pulse tube / Stirling produce 0.95 W @ 80 K / 0.32 W @ 35 K and 0.88 W @ 80 K / 0.29 W @ 35 K, respectively. In contrast, almost all the two-stage Stirling configuration effective refrigeration comes from the first stage (2.96 W @ 80 K / 0.13 W @ 35K). The two-stage Stirling design would have to be modified for most practical applications at the 35 K / 80 K temperature split, whereas the hybrid designs would not.

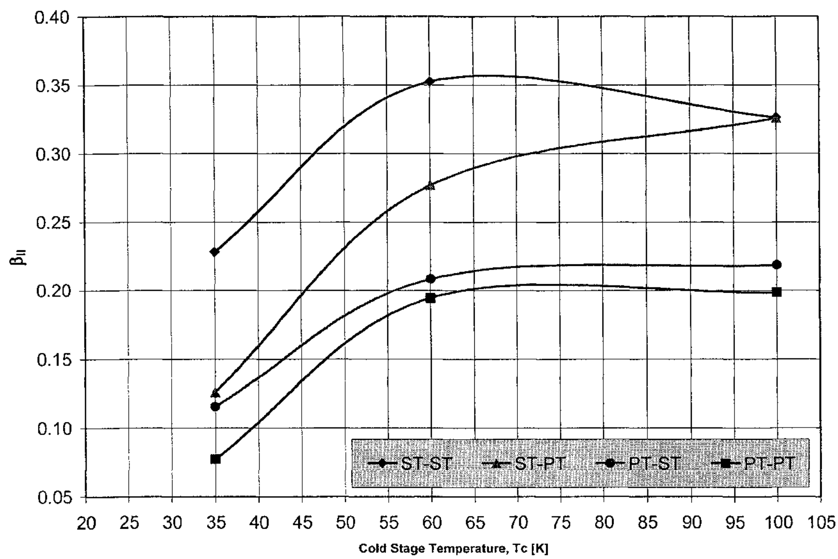

FIGURE 6. Second Law Efficiency for Two-Stage Cooler Configurations. ST-PT increasingly efficient at warmer temperatures. See Fig 5 for legend.

CASE STUDY: STIRLING / PULSE TUBE

A Stirling / pulse tube design was optimized for 1 W @ 35 K simultaneously with 5 W @ 80 K. That design was then exercised at constant operating conditions, including Stirling displacer phase angle, over a range of mid-stage temperatures from 50 K to 110 K. The results are shown in FIGURE 7. The efficiency is fairly constant from 75 K to 95 K, but it decreases rapidly outside that range at both the high and low end. Given the particularly poor performance exhibited at the mid-stage temperature extremes, the possibility of improving the performance at 50 K and 110 K by using the Stirling / pulse tube's load shifting capability was investigated. The optimum mechanical phase angle between the Stirling displacer piston and compressor piston for the 35 K / 80 K design point was determined to be $\phi_{opt} = 83.2^{o}$. The efficiency as a function of phase angle from $\phi_{opt} - 30^{o}$ to $\phi_{opt} + 30^{o}$ is plotted in Figure 8 for the 50 K and 110 K mid-stage temperatures of interest. The 80 K case is included for reference. The efficiency at 50 K is not improved through load shifting. The 110 K efficiency, however, is improved from 0.13 to 0.17 by increasing the phase angle 20° above ϕ_{opt}. In the same way the relatively high two-stage Stirling efficiency at 35 K / 80 K was misleading because the capacity was so heavily slanted towards the first stage, the first stage capacity at 103.2° is 10.75 W with only 0.14 W at the second stage. This is unlikely to be of practical utility for a real system, except for the case where the Stirling / pulse tube is coupled with a thermal storage unit at the first stage and load shifting is being used to efficiently manage a duty-cycled second-stage load [4]. Ultimately, the optimum operating point of the Stirling / pulse tube is a function of the entire cryogenic system, not just the cryocooler. The ability to load shift, however, does make it far more likely that a single cryocooler design will be able to efficiently manage a wide range of temperatures and loads.

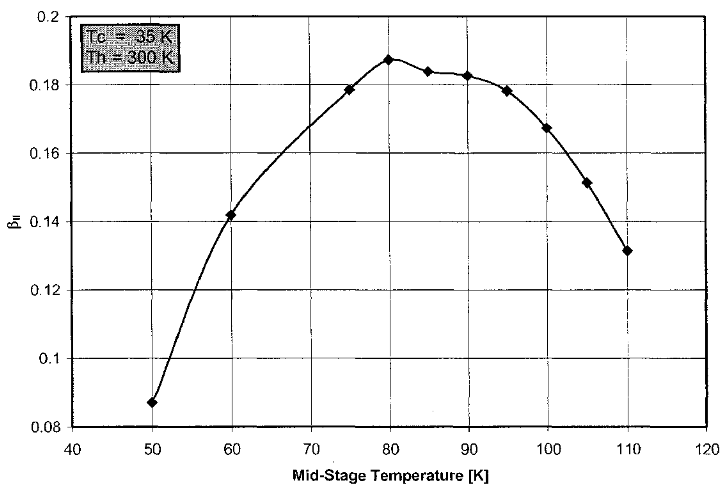

FIGURE 7. Second Law Efficiency of Stirling / Pulse Tube versus Mid-Stage Temperature. The efficiency rolls of quickly below the 80 K design point but is fairly flat out to 95 K.

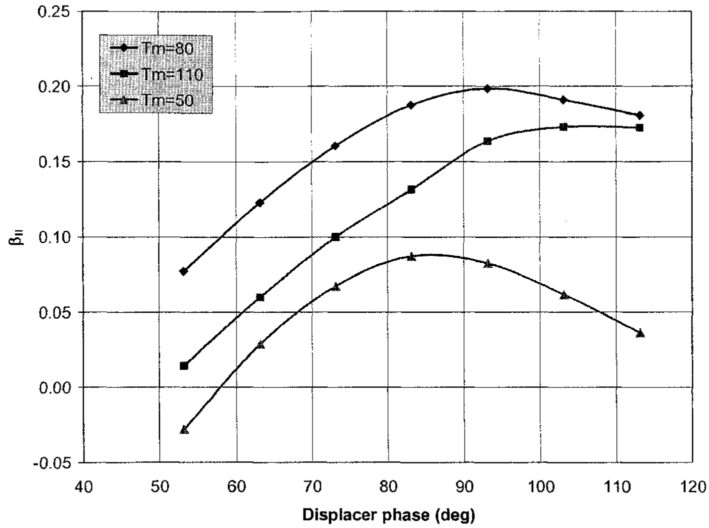

FIGURE 8. Second Law Efficiency versus Displacer Phase Angle.

CONCLUSION

In the comparative study, the best thermodynamic performance was achieved with the two-stage Stirling. The Stirling / pulse tube was next, and it has the advantages of a) having a less complex mechanical design because of the absence of a moving cryogenic seal and b) providing load shifting for more flexible operation.

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