Thermodynamic Optimization of Multi-Stage Cryocoolers

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ABSTRACT

Active Stirling class cryocoolers, including pulse tube coolers, are complex, difficult to optimize machines. The large number of characteristics and properties associated with geometry, materials, gas properties, heat transfer devices, flow manifolds, mechanical mechanisms, and electro-mechanical devices that determine a particular machine's performance make quick optimization difficult. Single-stage coolers are now sufficiently well understood that design optimization is reasonably straightforward. However, multi-stage coolers compound the design problem by virtue of the dramatically enlarged number of variables, and optimization is still a challenge. Often, multi-stage machines are "optimized" by a brute force search of the design space or design decisions are made based on overly generalized or inaccurate assumptions about relationships between variables. The schedule-constrained time typically available to perform optimization procedures combined with the large number of variables and their complex interaction results in sub-optimal products. This paper presents a concept for optimization that more rapidly converges on an optimal design.

INTRODUCTION

This paper presents a cryocooler design optimization method based on analysis of exergy flow. Exergy analysis follows the destruction of energy availability from the machine input to the low temperature stage(s) where refrigeration is produced.

First, the terms and approach of the exergy method are reviewed and clarified with reference to thermodynamic analysis of one-stage coolers. Then the analysis is extended to two-stage coolers. This is followed by a discussion of the optimization technique in one- and two-stage cryocoolers and, finally, the generalization of the method to multi-stage cryocoolers.

THERMODYNAMIC ANALYSIS

Single-Stage Cryocooler Thermodynamics

A careful examination of the thermodynamics of the comparatively simple single-stage cryocooler is helpful in describing the operation of the more complicated multi-stage devices. For that reason, a brief review of fundamental single-stage cryocooler thermodynamics is presented.

The conventional approach to cryocooler thermodynamic design involves the variation of design parameters, such as component volumes, regenerator matrix type and dimensions, etc., so

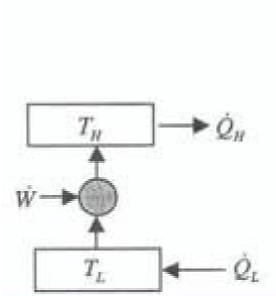

Figure 1. Energy flow schematic for single-stage refrigerator; arrow denotes energy flow direction.

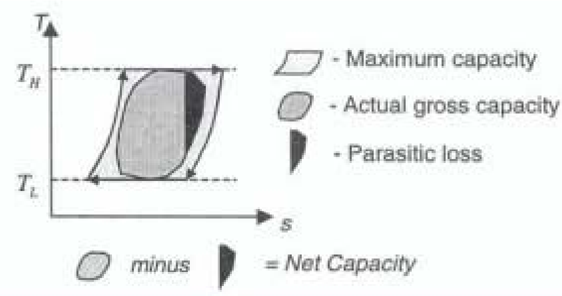

Figure 2. Temperature entropy diagram for a single-stage Stirling refrigerator.

that the capacity objective is achieved with the maximum thermodynamic efficiency. ARCOPTR is an example of a widely-used model for this type of optimization.[1] Though challenging in practice because of the complicated and interdependent physical phenomena involved, this approach is conceptually simple. Consider the energy flow diagram for a single-stage cooler shown in Figure 1. The conservation of energy dictates that

$$\dot{Q}_H = \dot{W} + \dot{Q}_L \tag{1}$$

and the coefficient of performance (COP) is given by

$$\beta = \frac{\dot{Q}_L}{\dot{W}} \tag{2}$$

The design parameters are therefore varied such that β is maximized for the desired $\dot{Q}_L$ at T_L. The effectiveness of this approach is hampered by an imperfect knowledge of the detailed internal thermodynamics and the vast trade space afforded by the multitudinous design options.

The primary shortcoming of the above approach is that it provides no figure of merit by which to evaluate the relative goodness of the β achieved. The standard figure of merit, the Carnot COP (β_c), is obtained by considering a reversible refrigerator operating between the temperature extremes of interest, and is shown in innumerable thermodynamics texts to be defined as follows:

$$\beta_c = \frac{\dot{Q}_{L,rev}}{\dot{W}} = \frac{T_L}{T_H - T_L} \tag{3}$$

A Second Law COP can now be defined in terms of Eqns. (2) and (3):

$$\beta_{II} = \frac{\beta}{\beta_c} \tag{4}$$

In words, Eq. (3) states that $\dot{Q}_{L,rev}$ is the maximum amount of refrigeration that can be obtained from a refrigerator operating between T_H and T_L given an input power of $\dot{W}$. This can be clearly shown on a temperature-entropy (T-s) diagram for a refrigerator operating on a Stirling cycle (Figure 2). The area bounded by the curves is the net refrigeration produced. The actual refrigeration is less than the maximum possible (reversible) refrigeration for a refrigerator operating between the given temperature extremes because of practical inefficiencies (irreversibilities) which can be roughly grouped into two categories, pneumatic losses and parasitic losses. Pressure drops, rounding of the corners of the T-s area due to real-world limits over phase control, seal leakage losses, and other such losses that decrease the amount of gross refrigeration produced are termed here "pneumatic losses." Conduction losses, radiative loads, regenerator inefficiency, and other losses that consume a portion of the gross refrigeration produced are differentiated into another category called "parasitic losses." Multiple expressions for the net refrigeration rate arise from these definitions:

$$\dot{Q}_L = \dot{Q}_{L,net} = \dot{Q}_{L,rev} - \dot{Q}_{lost} = \dot{Q}_{L,rev} - \dot{Q}_{pneumatic} - \dot{Q}_{parasitic} = \dot{Q}_{gross} - \dot{Q}_{parasitic} \quad (5)$$

Optimum refrigerator performance can also be sought through the method of entropy generation minimization. Referring back to Fig. 1, it is clear that the rate of entropy generation is given by

$$\dot{S}_{gen} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_L}{T_L} \quad (6)$$

Efficiency is maximized when $\dot{S}_{gen}$ is minimized. Though intuitively evident from these simple expressions, it is interesting to note that the equivalence of the First Law and Second Law methods can be demonstrated by considering the implications of irreversibilities on a refrigeration cycle from two distinct perspectives. Using the interpretation provided above that fixes the input power and attributes the difference between the reversible refrigeration rate and the actual refrigeration rate to the irreversibilities in the system, an expression can be obtained for the lost refrigeration in terms of the other system parameters:

$$\dot{Q}_{lost} = \left(1 + \beta_c^{-1}\right)\dot{Q}_{L,rev} - \dot{Q}_H \quad (7)$$

Using an alternative interpretation in which the refrigeration rate is fixed and the irreversibilities manifest as an increase in the actual required input power above that required for a reversible system, i.e.,

$$\dot{W}_{lost} = \dot{W} - \dot{W}_{rev} \quad (8)$$

a similar expression to Eq. (7) can be obtained for the lost input power:

$$\dot{W}_{lost} = \dot{Q}_H - \left(1 + \beta_c^{-1}\right)\left(\dot{Q}_{L,rev} - \dot{Q}_{loss}\right) \quad (9)$$

The algebraic combination of Eqns. (7) and (9) reduces to the simple expression

$$\dot{W}_{lost} = \frac{\dot{Q}_{lost}}{\beta_c} \quad (10)$$

Using the definition for lost work (power) provided in Bejan and elsewhere, this analysis finally yields to an expression for the lost refrigeration capacity in terms of the entropy generation:

$$\dot{Q}_{lost} = \beta_c T_0 \dot{S}_{gen} \quad (11)$$

This expression demonstrates that the First Law optimization approach of minimizing lost refrigeration capacity is functionally equivalent to minimizing the total rate of entropy generation, whatever the source. (The reference temperature T_0 can be taken as T_H for a refrigerator rejecting waste heat to the ambient environment.)

All of these concepts of energy flow and irreversibility come together in the exergy flow map provided in Figure 3, which is modeled after the techniques demonstrated by Bejan.[2] The input exergy, also called availability, provided by the compressor flows down from the warm end reservoir T_H to the refrigeration temperature T_L where refrigeration is produced. Exergy is destroyed through the pneumatic and parasitic loss mechanisms. The exergy content of a power interaction is equal to the power, while the exergy associated with a heat transfer interaction is a function of the temperature at which the heat transfer occurs:

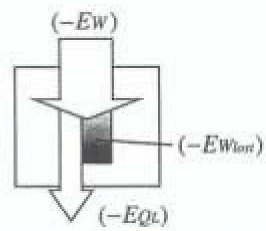

Figure 3. Exergy flow diagram for a single-stage refrigerator. Exergy is destroyed through parasitic and pneumatic losses.

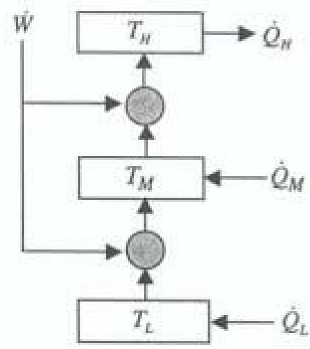

Figure 4. Energy flow diagram for a particular type of two-stage refrigerator. Other two-stage configurations may dictate alternative energy flow diagrams.

$$\dot{E}_W = \dot{W}$$
$$\dot{E}_{Q,i} = \dot{Q}_i\left(1 - \frac{T_O}{T_i}\right) \tag{12a,b}$$

Thus the rejection of heat to ambient ($T_H = T_O$) carries with it zero exergy. A refrigerator's minimum achievable "no load" temperature occurs at the temperature where all of the input exergy has been destroyed through irreversible losses. Of note is the fact that the distribution of losses between pneumatic and parasitic is immaterial from the standpoint of optimization through entropy generation minimization for single-stage refrigerators. As shown in the next section, this is not the case for multi-stage devices.

Two-Stage Cryocooler Thermodynamics

The thermodynamic analysis and optimization of a two-stage refrigerator follows directly from the analytical approach used for a single-stage device. The stages of multi-stage refrigerators can be arranged thermodynamically in many ways; an energy flow schematic for the type of staging of present interest, a single ambient compressor with the coldest stage rejecting heat to the intermediate stage, is provided in Figure 4. The conservation of energy equation is again defined at the boundaries of the refrigerator and is given by

$$\dot{Q}_H = \dot{W} + \dot{Q}_M + \dot{Q}_L \tag{13}$$

The input power that drives both stages originates at T_H, from the ambient compressor. Therefore, the definitions for both the first stage and second stage Carnot COPs are based upon the ambient temperature T_H:

$$\beta_{c,L} = \frac{T_L}{T_H - T_L}$$
$$\beta_{c,M} = \frac{T_M}{T_H - T_M} \tag{14a,b}$$

The standard energy flow diagram used in Figure 4 is somewhat misleading in that it appears to show an engine residing between T_M and T_L, and this leads to the temptation to use T_M in Eq.

(14a). This is incorrect for the single compressor problem. If one were to use T_M, it can be shown that the perfectly reversible system yields negative entropy generation, not zero, which violates the Second Law of Thermodynamics. In contrast, if a multi-stage refrigerator were to be considered in which individual compressors operating between each temperature level were used to pump heat between those adjacent temperature reservoirs, then the Carnot COP definitions would be based upon the temperatures bounding each stage.

The input power can be conceptually partitioned into the individual portions required to drive each stage. The reversible input power can thus be defined as

$$\dot{W}_{1,rev} = \frac{\dot{Q}_M}{\beta_{c,M}} \qquad (15a,b)$$
$$\dot{W}_{2,rev} = \frac{\dot{Q}_L}{\beta_{c,L}}$$

where $\dot{Q}_M$ and $\dot{Q}_L$ are the actual net refrigeration rates. The reversible input power for each stage represents the minimum theoretical power required to deliver the prescribed refrigeration rate between T_H and the refrigeration temperature. The irreversible, or lost, power can be similarly partitioned, and the approach demonstrated for the single-stage cooler in developing Eq. (10) leads to similar looking expressions for the two-stage device:

$$\dot{W}_{1,lost} = \frac{\dot{Q}_{M,lost}}{\beta_{c,M}} \qquad (16a,b)$$
$$\dot{W}_{2,lost} = \frac{\dot{Q}_{L,lost}}{\beta_{c,L}}$$

The input power lost due to irreversibilities in the first stage represents an overall system loss, but some of the second stage loss, in particular that portion due to parasitic losses between T_M and T_L, is partially recoverable. The energy transfer from T_M to T_L through conduction losses and regenerator enthalpy flux due to heat exchanger inefficiency decreases the capacity at the second stage, but it increases the capacity at the first stage by the same amount. Therefore, the parasitic loss portion of $\dot{W}_{2,lost}$ is partially recovered:

$$\dot{W}_{2,rec} = \frac{\dot{Q}_{L,parasitic}}{\beta_{c,M}} \qquad (17)$$

Note that the recoverable fraction of the second-stage power lost due to parasitics is given by the ratio of the Carnot efficiencies, $\beta_{c,L}/\beta_{c,M}$.

The thermodynamic intricacies of the two-stage cooler analysis are captured in the exergy flow map provided in Figure 5. The flow of input exergy from ambient and its partitioning between the refrigeration stages illustrates the proper selection of T_H in Eq. (14a) as the warm end reference temperature. The map also shows that the second stage parasitic loss carries with it positive exergy, that a portion is destroyed in the transferring of capacity from the colder stage to the intermediate stage, and that the remainder represents the recoverable exergy. The recoverable exergy can be shown to be equal to the recoverable power defined in Eq. (17) by considering the net exergy change due to the second stage parasitic loss component:

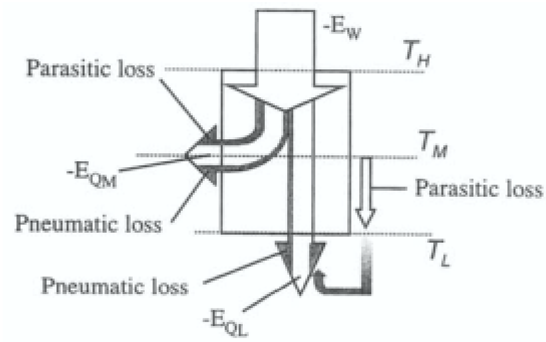

Figure 5. Exergy flow map for two-stage cryocooler. Parasitic heat transfer from middle stage to cold stage increases the capacity at T_M and decreases the capacity at T_L by the same amount. This represents a net exergy loss to the system.

$$\Delta E_{Q_{L,parasitic}} = -\dot{Q}_{L,parasitic}\left(1-\frac{T_H}{T_M}\right)-(-\dot{Q}_{L,parasitic})\left(1-\frac{T_H}{T_L}\right) \tag{18}$$

The right hand side can be expressed in terms of the Carnot efficiencies:

$$\Delta E_{Q_{L,parasitic}} = -\dot{Q}_{L,parasitic}\left(\frac{-1}{\beta_{c,M}}\right)-(-\dot{Q}_{L,parasitic})\left(\frac{-1}{\beta_{c,L}}\right) \tag{19}$$

This expression reduces to

$$\Delta E_{Q_{L,parasitic}} = \left(\frac{\dot{Q}_{L,parasitic}}{\beta_{c,M}}\right)-\left(\frac{\dot{Q}_{L,parasitic}}{\beta_{c,L}}\right) \tag{20}$$

where the second term on the right hand side represents the exergy destruction and the first term is the recovered exergy. Note that the right hand side is always negative; although there is partial recovery, some exergy is destroyed through the second stage parasitic loss.

CHARACTERIZATION AND OPTIMIZATION TECHNIQUES

Single-Stage Cryocoolers

Thermodynamic efficiency is maximized when the rate of entropy generation is minimized. The optimization of single-stage cryocoolers through numerical analysis typically involves the variation of design parameters such as component size, operating frequency, and phase angles in search of a maximum refrigeration capacity-to-input power ratio. This is simply an indirect method of minimizing entropy generation. For a typical design study, the refrigeration load, refrigeration temperature, and nominal heat rejection temperature are known, and the input power required to deliver the needed capacity is being minimized. Referring to the expression for entropy generation in Eq. (6), the only unknown on the right hand side is $\dot{Q}_H$, so entropy generation is minimized when the heat rejection rate is minimized. The reversible input power is fixed by the refrigeration load and the temperature extremes, so $\dot{W}_{rev}$ is known as well. Combining Eqns. (1) and (8) yields

$$\dot{Q}_H = \dot{W}_{lost} + \dot{W}_{rev} + \dot{Q}_L \tag{21}$$

in which only $\dot{W}_{lost}$ is unknown. Therefore, the task reduces to the minimization of $\dot{W}_{lost}$.

The intent of introducing these Second Law considerations is not to replace the parametric characterization of design variables in cryocooler optimization, rather the purpose is to improve the parametric studies by guiding the efforts through an improved understanding of the physics of the system. The Second Law techniques are particularly useful for characterizing the efficiency of the cryocooler using β_{II} as a figure of merit. For example, if a cryocooler is sufficiently well understood to express the losses in terms of known system parameters, then β_{II}, determined from the entropy generation calculations, can be used to describe the efficiency of the cryocooler over a range of refrigeration temperatures. Assume the refrigeration losses can be approximated by an expression of the form

$$\dot{Q}_{lost} = \alpha_1(\bar{T}) * (T_H - T_L)^{\alpha_2(\bar{T})} \tag{22}$$

where

$$\bar{T} = (T_H + T_L)/2 \tag{23}$$

and $\alpha_1(\bar{T})$ and $\alpha_2(\bar{T})$ are known. (Ideally, the cryocooler is designed such that α_1 is minimized in the vicinity of the design point corresponding to $\bar{T}$.) From Eqns. (3) and (10), the total input power is given by

$$\dot{W} = \frac{1}{\beta_c}\left(\dot{Q}_L + \dot{Q}_{lost}\right) \tag{24}$$

The Second Law COP follows immediately from the above using Eqns. (2) and (4). Examples of where this type of characterization might prove useful include design efforts to rescale an existing cryocooler to increase capacity and thermal system trade studies in which various cryocoolers are being considered for applications outside their originally intended and better characterized range of operation.

Two-Stage Cryocoolers

The increased thermodynamic complexity of a multi-stage cryocooler gives rise to a substantially more challenging task of parametric characterization than for a single-stage cooler, thus making the practical application of Second Law techniques all the more valuable. Consider a two-stage cryocooler design in which the refrigeration loads and temperatures at both stages are prescribed together with the heat rejection temperature. As described above, entropy generation minimization is equivalent to reducing the total lost input power, which is given by the algebraic combination of Eqns. (16a), (16b), and (17):

$$\dot{W}_{TTL,lost} = \dot{W}_{1,lost} + \dot{W}_{2,lost} - \dot{W}_{2,rec} \tag{25}$$

Substituting and combining terms yields

$$\dot{W}_{TTL,lost} = \frac{1}{\beta_{c,M}}\left(\dot{Q}_{M,lost^*} - \dot{Q}_{L,parasitic^*}\right) + \frac{\dot{Q}_{L,lost}}{\beta_{c,L}} \tag{26}$$

Expressions are needed for the lost refrigeration in the form of the general equation

$$\dot{Q}_{i,lost} = f_i(T_L, T_M, T_H) \tag{27}$$

and for the second-stage parasitic loss

$$\dot{Q}_{L,parasitic} = g(T_L, T_M) \tag{28}$$

The above expressions assume the parasitic loss between the intermediate and coldest stage is driven by the temperature difference between those stages, as would be expected for a conductive or convective loss, but allows for the possibility of more complicated interactions between the stages in determining the total loss, which includes the pneumatic loss component. However, since the adjacent boundary temperatures likely dominate the total loss between stages, Eq. (27) can be simplified:

$$\begin{aligned} \dot{Q}_{L,lost} &= f_L(T_L, T_M) \\ \dot{Q}_{M,lost} &= f_M(T_M, T_H) \end{aligned} \tag{29a,b}$$

As an example, consider an application in which a two-stage cryocooler design is being considered to meet a single prescribed refrigeration load and temperature. No intermediate refrigeration capacity is required, so the temperature at the intermediate stage is immaterial to the user. Given this flexibility, the cryocooler designer seeks the optimum intermediate temperature such that the design capacity at the coldest stage is achieved with minimum entropy generation. Using a simplified form of the loss equation from Eq. (22) in which it is assumed any non-linearities with respect to temperature are captured in the first coefficient, the following expressions for total lost refrigeration are obtained:

$$\begin{aligned} \dot{Q}_{L,lost} &= \alpha(\overline{T}_{L->M}) * (T_M - T_L) \\ \dot{Q}_{M,lost} &= \gamma(\overline{T}_{M->H}) * (T_H - T_M) \end{aligned} \tag{30a,b}$$

For the purposes of this example, assume the parasitic loss is a linear function of the temperature difference (conduction and convection dominated):

$$\dot{Q}_{L,parasitic} = \kappa * (T_M - T_L) \tag{31}$$

The optimum intermediate temperature occurs where the total lost input power (Eq. 26) is minimized, i.e., where

$$\begin{aligned} &\frac{\partial}{\partial T_M}(\dot{W}_{TTL,lost}) = 0 \\ &\frac{\partial^2}{\partial T_M^2}(\dot{W}_{TTL,lost}) > 0 \end{aligned} \tag{32a,b}$$

The partial derivative of interest is calculated through several sequential applications of the chain rule, only a few of which are shown below for the sake of conciseness.

$$\begin{aligned} &\frac{\partial}{\partial T_M}(\dot{W}_{TTL,lost}) \\ &= \frac{\partial}{\partial T_M}\left(\frac{1}{\beta_{c,M}}(\dot{Q}_{M,lost} - \dot{Q}_{L,parasitic}) - \frac{\dot{Q}_{L,lost}}{\beta_{c,L}}\right) \\ &= \frac{1}{\beta_{c,M}}\frac{\partial}{\partial T_M}(\dot{Q}_{M,lost} - \dot{Q}_{L,parasitic}) - (\dot{Q}_{M,lost} - \dot{Q}_{L,parasitic})\frac{\partial}{\partial T_M}\left(\frac{1}{\beta_{c,M}}\right) + \frac{1}{\beta_{c,L}}\frac{\partial}{\partial T_M}(\dot{Q}_{L,lost}) + \dot{Q}_{L,lost}\frac{\partial}{\partial T_M}\left(\frac{1}{\beta_{c,L}}\right) \\ &= \frac{1}{\beta_{c,M}}\frac{\partial}{\partial T_M}(\dot{Q}_{M,lost} - \dot{Q}_{L,parasitic}) - (\dot{Q}_{M,lost} - \dot{Q}_{L,parasitic})\left(\frac{-T_H}{T_M^2}\right) + \frac{1}{\beta_{c,L}}\frac{\partial}{\partial T_M}(\dot{Q}_{L,lost}) \end{aligned} \tag{33}$$

The chain rule is applied again for each loss component. For example,

$$\frac{\partial}{\partial T_M}\left(\dot{Q}_{M,lost}\right) = \frac{\partial}{\partial T_M}\left(\gamma(T_H - T_M)\right) = -\gamma + \frac{\partial \gamma}{\partial T_M}(T_H - T_M) \tag{34}$$

The functions α and γ and the second-stage parasitic loss constant κ are assumed to be known, which is consistent with the premise that the performance characteristics of each stage is reasonably well understood, and it is the optimum combination of those stages into a single device which is sought. Eventually, through substitution and further application of the chain rule, the lost power derivative in Eq. (33) is reduced to a singular expression in terms of the system's one unknown, T_M. The derivative can then be calculated over the temperature range of interest and the optimum value for T_M thus identified.

DISCUSSION

Multi-Stage Cryocooler Figure of Merit

The application of Second Law principles to cryocooler thermodynamic design helps guide the design by providing a figure of merit that relates performance to that of an ideal cryocooler. Eq. (4) defines that figure of merit, β_{II}, for a single-stage cryocooler. For two-stage and other multi-stage cryocoolers, the concept of exergy flow can be used to define similar figures of merit. As shown in Eq. (12b), the exergy associated with a heat transfer interaction between a system and its surroundings is proportional to the inverse of the Carnot COP corresponding to the temperature at which the heat transfer process occurs. For a cryocooler, the refrigeration capacities occurring at various temperature levels can be normalized to a single refrigeration capacity at a single arbitrary temperature, and the resulting normalized efficiency can then be compared to that of a single-stage Carnot refrigerator. Typically, the temperature at the coldest stage is used to normalize the capacities because it is the achievement of positive refrigeration at that temperature level which stresses the design. It follows that the normalized "pseudo single stage" refrigeration for an *n*-stage refrigerator is given by the expression

$$\dot{Q}_L^{norm} = \sum_{i=1}^{n} \frac{\beta_{c,L}}{\beta_{c,i}}\left(\dot{Q}_i\right) \tag{35}$$

The COP and Second Law efficiency definitions follow directly from Eqns. (2), (4), and (35):

$$\beta^{norm} = \frac{\dot{Q}_L^{norm}}{\dot{W}} \tag{36}$$

$$\beta_{II}^{norm} = \frac{\beta^{norm}}{\beta_{c,L}} \tag{37}$$

These expressions are useful in comparing multi-stage cryocoolers to each other and to single-stage cryocoolers, and they can also be used to evaluate the relative efficiency between varied temperature distributions and heat loads for a multi-stage cryocooler operating. The latter is essentially a generalized extension of the two-stage example used above where the net intermediate refrigeration is allowed to vary from zero.

Extension of Exergy Flow Concepts to *n*-Stage Cryocoolers

The thermodynamic analysis of cryocoolers with more than two cold stages follows directly from the two-stage cryocooler analysis. Assuming a single ambient compressor is driving the multi-stage expander, the Carnot efficiency for each stage is based upon the ambient temperature and the refrigeration temperature for that stage, i.e.,

$$\beta_{c,i} = \frac{T_i}{T_H - T_i} \ , \quad i = 1.....n \tag{38}$$

The concepts of lost power at each stage, the partial recovery of exergy flow due to parasitic heat transfer losses, and the minimization of total lost power for maximum efficiency all translate directly from the two-stage cryocooler analysis. The number of terms in the total lost power expression (Eq. 25) grows with *n*, so the calculation of the partial derivatives required to characterize the efficiency of the cryocooler over the range of interest becomes more complicated, but the fundamental approach is the same. The value of employing these techniques actually increases as the number of refrigeration stages increases because the voluminous trade space associated with multi-stage cryocoolers becomes nearly impossible to adequately characterize with brute force parametric design trades.

CONCLUSION

The introduction of exergy flow concepts into the thermodynamic analysis of cryogenic refrigerators provides a clear illustration of how internal irreversibilities, which destroy exergy, are manifested at the system boundary as decreased performance. The decreased performance can be interpreted as either a loss in refrigeration capacity for a given input power or an increase in the required input power for a desired refrigeration capacity. The Second Law optimization approach of minimizing entropy generation (i.e., exergy destruction) is shown to yield particular advantage for multi-stage refrigerators because the trade space is broad and the thermodynamic interactions complex. Interestingly, Streich reached a similar conclusion in his exergy analysis of a quite different problem, the thermodynamic characterization of a mixing process involving two natural gas streams.[3] He concluded, in part, that exergy analysis is particularly useful for "pioneering" and "systematic studies," terms which aptly describe the exercise of developing and optimizing a multi-stage cryogenic refrigerator.

Future work is planned in which quantitative loss correlations for a multi-stage cryocooler will be substituted into the analytical model described herein to demonstrate the proposed Second Law optimization approach.

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