LONGITUDINAL HYDRAULIC RESISTANCE PARAMETERS OF CRYOCOOLER AND STIRLING REGENERATORS IN PERIODIC FLOW

Cite as: AIP Conference Proceedings **985**, 259 (2008); https://doi.org/10.1063/1.2908555
Published Online: 27 March 2008

J. S. Cha, S. M. Ghiaasiaan, and C. S. Kirkconnell

AIP Conference Proceedings **985**, 259 (2008); https://doi.org/10.1063/1.2908555 **985**, 259

LONGITUDINAL HYDRAULIC RESISTANCE PARAMETERS OF CRYOCOOLER AND STIRLING REGENERATORS IN PERIODIC FLOW

J. S. Cha[1], S. M. Ghiaasiaan[1], C.S. Kirkconnell[2]

[1]G.W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology Atlanta, Ga, 30332, USA

[2]Raytheon Space and Airborne Systems (RSAS)
El Segundo, Ca, 90245, USA

ABSTRACT

The results of an on going research program aimed at the measurement and correlation of anisotropic hydrodynamic parameters of widely-used cryocooler regenerator fillers are presented. The hydrodynamic parameters associated with longitudinal periodic flow are addressed in this paper. An experimental apparatus consisting of a cylindrical test section packed with regenerator fillers is used for the measurement of axial permeability and Forchheimer coefficients, with pure helium as the working fluid. The regenerator fillers that are tested include stainless steel 400-mesh screens with 69.2% porosity, stainless steel 325-mesh screens with 69.2% porosity, stainless steel 400-mesh sintered filler with 62% porosity, stainless steel sintered foam metal with 55.47% porosity, and nickel micro-machined disks with 26.8% porosity. The test section is connected to a Stirling type compressor on one end and to a constant volume chamber on the other end. The instrumentation includes piezoelectric pressure transducers at both ends of the regenerator and a hot wire anemometer at the inlet of the regenerator. For each filler material, time histories of local pressures at both ends of the regenerator are measured under steady periodic conditions over a wide range of oscillation frequencies (5 - 60 Hz). A CFD assisted methodology is then used for the analysis and interpretation of the measured data. The viscous resistance coefficient and the inertial resistance coefficient values obtained in this way are correlated in terms of the relevant dimensionless parameters.

KEYWORDS: Porous media, longitudinal (axial) permeability for periodic flow, Helium

INTRODUCTION

CP985, *Advances in Cryogenic Engineering: Transactions of the Cryogenic Engineering Conference—CEC, Vol. 53,* edited by J. G. Weisend II

The regenerator is a key component in all Pulse Tube Refrigerator (PTR) designs. The PTR regenerator is typically a complex porous structure and ideally would have infinite thermal inertia, infinite volumetric heat transfer coefficients with the working fluid, negligible pressure drop, and negligible thermal conductivity and thermal dispersion in the axial direction. In reality, however, compromise between pressure drop and heat transfer is required for optimization. It has recently been shown that computational fluid dynamic (CFD) tools can simulate entire PTR devices [1-4], and can be useful tools for the final stages of design and optimization. CFD simulations provide details that are beyond the current experimental measurement possibilities. However, such detailed predictions are only reliable if they are based on correct closure relations which can only be obtained through integral experimental measurements.

The hydrodynamic parameters for many regenerator fillers have been measured and published in the open literatures. However, they are mostly based on steady flow experiments, and only a limited number of investigations dealing with oscillatory hydrodynamic parameters have been recently published [5-7].

In this paper, we report on periodic flow experiments aimed at the measurement of longitudinal (axial) permeability and coefficient of inertia (Forchheimer's coefficient) for various regenerator fillers under steady periodic flow for frequency range of 5 Hz to 60 Hz.

EXPERIMENTAL APPARATUS

The test apparatus (Figure 1) includes a function generator, a data acquisition system, drive electronics (amplifier and function generator), a compressor (4.29 cc swept volume Hughes Tactical Condor), three PCB piezo pressure transducers, a constant temperature hot wire anemometer, a buffer volume, and a specially designed regenerator module. Research grade Helium (nominal purity of 99.9999%) was used. Time histories of local instantaneous pressures at the inlet and exit of the regenerator were measured. The regenerator inlet velocity was also measured by Hot Wire Anemometry (HWA).

Two regenerator test modules, RTS1 (7.94 mm I.D. and 38.1 mm long) and RST2, were designed and fabricated. RTS1 (Figure 2) has a housing module with flange type end-connections, Viton O-ring seals, and two flange type connecting components (7.62 mm I.D. and 40.6 mm long). RTS2 section was similar to RTS1, except that it had a larger diameter and shorter length. Its inner diameter and length were 14.99 mm and 31.4 mm, respectively. Its end connecting components had an inner diameter and length of 12.7 mm and

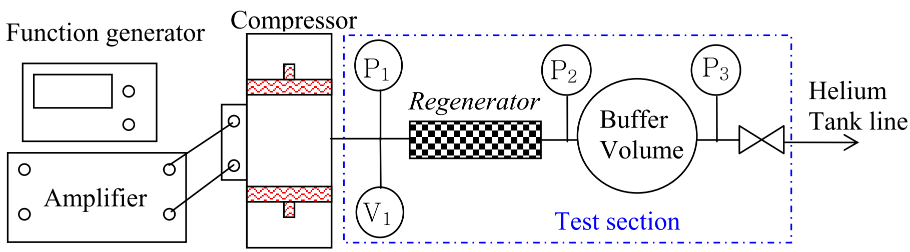

FIGURE 1. Experimental apparatus for oscillatory flow pressure drop and regenerator inlet velocity measurement

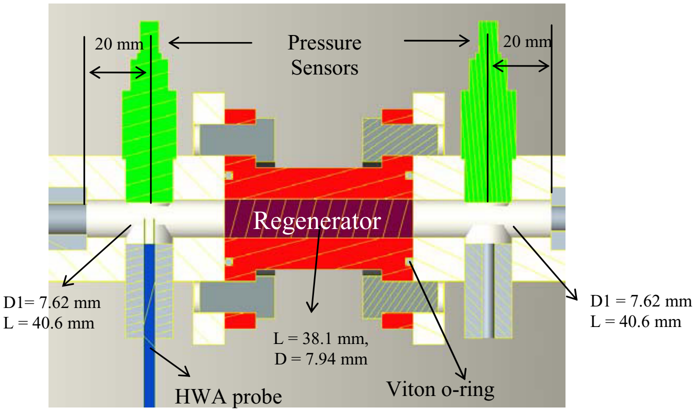

FIGURE 2. Detail description of RTS1 regenerator test section.

19.05 mm, respectively. Three PCB peizo transducers (model A101A05, from PCB electronics, less than 2 μsec response time, 0 ~689 kPa range, 0.014 kPa resolution, 7.3 mV/kPa sensitivity) measured the local instantaneous pressures. A constant temperature hot wire anemometer was used to measure the local instantaneous velocities at the inlet of the regenerator, V_1, and it was calibrated using steady flow test loop with mass flow meter (Toptrack Model 1500) at room temperature.

Five regenerator samples were tested (Table 1). Four samples [fine wire mesh screens (325 and 400 mesh), sinter 400 mesh, and metallic foam metal] were tested using RTS1. However, the stacked nickel micro-machined disks (36-40 μm perforation diameters) were tested using RTS2. Seven oscillatory longitudinal pressure drop tests were conducted with each filler covering 5 to 60 Hz. Each test had a fixed compressor frequency. In six of the tests (excluding a test at 5 Hz) the peak to peak sinusoidal voltage amplitude was first increased via the function generator until either the maximum compressor piston displacement or the maximum current limit were reached. The voltage amplitude was then maintained constant and the steady-periodic pressures at P_1, P_2, and P_3 and velocities at V_1 (see Figure 1) were recorded. For the 5 Hz frequency low flow conditions were sought so that the permeability in Darcy flow conditions could be measured, therefore the peak to peak sinusoidal voltage amplitude was increased only sufficiently to ensure that pressure sensor signals were viable. To simplify the analysis, the recorded data were first transformed to the frequency domain by Fast Fourier Transforms (FFT), and were thereby represented as Fourier Cosine series. The first three harmonics were found to be sufficient

TABLE 1. Characteristics of the tested regenerator fillers.

Regenerator Type	Length/ Diameter [mm]	Wire diameter/ Pore diameter	Porosity [%]	Material
325 mesh screen	38.1 / 7.94	35.6 μm	69.2	Stainless Steel
400 mesh screen	38.1 / 7.94	25.4 μm	69.2	Stainless Steel
Sintered 400 mesh	38.1 / 7.94	sintered	62	Stainless Steel
Foam metal	38.1 / 7.94	sintered	55.47	Stainless Steel
Micro-machined Disks*	31.4 / 15	36-40 μm	26.8	Nickel

*provided by International Mezzo Technologies Inc. Baton Rouge, Louisiana.

for the accurate replication of the actual measured waveforms. The measured steady periodic pressures could thus be represented as:

$$P_i(t) = \Gamma_1 \cos(\Omega_1 t + \Delta_1) + \Gamma_2 \cos(\Omega_2 t + \Delta_2) + \Gamma_3 \cos(\Omega_3 t + \Delta_3)$$
$$\Omega_n = n\omega, \quad \omega = 2\pi f, \quad n = 1, 2 \qquad (1)$$

where $i = 1$ and 2. Table 2 is a summary of all the parameters in the above equation. More details about the experiments can be found in Cha [4].

CFD MODEL

Fluent ® [8] was used to model the entire test section, shown in Figure 2, in order to facilitate the interpretation of the experimental data. Axi-symmetric, two-dimensional flow was assumed. The simulated system evidently has two completely different parts. For the open parts, the steady state mass, momentum and energy equations solved by Fluent are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \qquad (2)$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u}\vec{u}) + \nabla P - \nabla \cdot (\overline{\overline{\tau}}) - \rho \vec{g} = 0 \qquad (3)$$

$$\nabla \cdot (k \nabla T + \overline{\overline{\tau}} \cdot \vec{u}) - \frac{\partial (\rho E)}{\partial t} - \nabla \cdot (\vec{u} \langle \rho E + P \rangle) = 0 \qquad (4)$$

where

$$E = h - p/\rho + u^2/2 \qquad \overline{\overline{\tau}} = \mu\left(\left(\nabla \vec{u} + \nabla \vec{u}^T\right) - (2/3)\nabla \cdot \vec{u}\overline{\overline{I}}\right) \qquad (5)$$

These equations apply to all sections except for the regenerator filler. The latter region is modeled as a porous medium with local thermal equilibrium assumption. The time dependent mass, momentum, and energy equations for this region can be represented as

$$\frac{\partial (\varepsilon \rho)}{\partial t} + \nabla \cdot (\varepsilon \rho \vec{u}) = 0, \qquad (6)$$

$$\frac{\partial (\varepsilon \rho \vec{u})}{\partial t} + \nabla \cdot (\varepsilon \rho \vec{u}\vec{u}) + \varepsilon \nabla P + \nabla \cdot (\varepsilon \overline{\overline{\tau}}) = \varepsilon \vec{F}_{bf} - \left\langle \frac{\mu}{\overline{\overline{\beta}}} \cdot \vec{u} + \frac{\overline{\overline{C}}\rho}{2} \cdot |\vec{u}|\vec{u} \right\rangle, \qquad (7)$$

$$\nabla \cdot \left[\left(\varepsilon k_f + (1-\varepsilon)k_s\right)\nabla T + \left(\overline{\overline{\tau}} \cdot \varepsilon \vec{u}\right)\right] = \frac{\partial}{\partial t}\left(\varepsilon \rho_f E_f + (1-\varepsilon)\rho_s E_s\right) + \nabla \cdot \left(\varepsilon \vec{u}\left(\rho_f E_f + P\right)\right). \qquad (8)$$

Here $\vec{u}$ represents the volume-averaged intrinsic (physical) fluid velocity, namely $\langle \vec{u} \rangle^f$.

Periodic flow CFD simulations for the entire test section depicted in Figure 1 were performed using approximately 9000 mesh nodes. The boundary conditions included known inlet instantaneous static pressure as a periodic function of time.

The porous regenerator is axi-symmetric, and its axial direction is a principle direction. Therefore, for axial flow the coefficients in the last two terms of Eq. (7) are related to Darcy permeability, K_x and Forchheimer's inertial coefficient, $c_{f,x}$, by [4]:

$$K_x = \varepsilon^2 \beta_x \qquad c_{f,x} = \frac{C_x \sqrt{K_x}}{2\varepsilon^3} \tag{9}$$

An alternative to these definitions is the friction factor defined according to

$$0.5 \frac{f_x}{\sqrt{\beta_x}} \rho |\vec{u}| u_x = \frac{\mu}{\beta_x} u_x + \frac{C_x \rho}{2} |\vec{u}| u_x \tag{10}$$

By non-dimensionalizing Eq. (10) the friction factor can be recast as:

$$f_x = \frac{2}{\mathrm{Re}_{\beta_x}} + C_x \sqrt{\beta_x} \qquad \mathrm{Re}_{\beta_x} = \frac{\rho |\vec{u}| \sqrt{\beta_x}}{\mu} \tag{11}$$

where Re_{β_j} is the local Reynolds number.

RESULTS AND DISCUSSION

Simulations were performed for all the measured data. A User Defined Function (UDF) was developed for Fluent, whereby for each frequency Eq. (1) with parameters representing P_1 (shown in Table 2), was applied as the inlet boundary condition. The

TABLE 2. Experimentally measured axial pressure drop for metal foam regenerator under oscillating flow.

Freq (Hz)	5	10	20	30	40	50	60
P_1							
Γ_1, [Pa]	35974.72	32697.21	39487.63	117866.12	70639.06	48387.14	47183.37
Γ_2, [Pa]	216.49	312.11	458.12	1176.50	180.94	199.07	248.96
Γ_3, [Pa]	1162.28	970.78	2232.79	8121.91	4854.68	3191.80	2974.19
Δ_1, [Deg]	-94.15	-104.21	-113.84	-123.02	-136.06	-148.47	-159.06
Δ_2, [Deg]	-167.52	154.87	86.50	86.17	-14.40	-111.64	-174.55
Δ_3, [Deg]	-136.41	-170.11	105.19	31.39	-32.37	-85.70	-132.92
P_2							
Γ_1, [Pa]	34557.92	27614.24	22675.08	36920.53	19120.44	11451.10	9163.25
Γ_2, [Pa]	281.54	191.77	141.32	261.78	76.28	38.30	35.23
Γ_3, [Pa]	787.05	196.17	82.93	245.44	90.19	27.91	20.29
Δ_1, [Deg]	-109.76	-135.44	-167.94	166.66	150.90	136.30	123.32
Δ_2, [Deg]	153.97	104.65	38.72	52.61	36.83	-4.71	-27.46
Δ_3, [Deg]	161.02	54.33	-55.01	150.94	137.23	98.83	-102.11

objective was to obtain the longitudinal permeabilities and coefficients of inertia by a series of rational steps. To do this, first, the case of 5 Hz which had the lowest experimental pressure drop was simulated by iteratively adjusting β_x without including the inertial effect (i.e., $C_x = 0$) until the simulation predictions for P_2 matched the experimental data. The 30 Hz case which had the largest experimental pressure drop was then simulated. This time only C_x was iteratively adjusted while β_x was kept constant until good agreement was obtained between the P_2 predictions and experimental data. Using these values of β_x and C_x simulations were then performed for all frequencies. If good agreement was obtained for all frequencies then iterative simulations would end, otherwise β_x and C_x would be iteratively adjusted to obtain the best possible match with all the experiment data. However, at this point only minor adjustments were needed. Using this methodology the parameters shown in Table 3 were obtained. Typical comparisons between the predicted and measured pressures at P_2 are displayed in Figures 3 and 4. Typical results comparing the predicted and measured mass flow rates are shown in Figure 5, where excellent agreement between the data and simulation is noted.

Figure 6 shows the ratio between the friction factors representing oscillatory and steady flows (f_{osc}/f_{steady}) for all the tested regenerator samples. The real data in these figures are depicted with symbols, and the dotted curves represent extrapolations based on the derived correlations for steady-flow and periodic-flow friction factors. This ratio evidently quantifies the deviation of oscillatory flow friction value from the steady flow friction value. For the Reynolds numbers up to about 0.1 the ratio is essentially one, and no significant deviations were observed and the steady friction factor values were virtually identical to the oscillatory friction factor. However, significant deviations can be noted at

TABLE 3. Oscillatory flow longitudinal (axial) hydrodynamic parameters.

Regenerator Type	β_x,[m^2]	C_x,[m^{-1}]	K_x,[m^2]	$c_{f,x}$,[-]	Porosity ε, [%]	Charge pressure [MPa]
325 mesh screens	6.4247e-11	67000	3.077e-11	0.561	69.2	2.78
400 mesh screens	2.5295e-11	120000	1.211e-11	0.630	69.2	2.78
400 mesh sintered	1.9828e-11	110000	7.622e-12	0.637	62	2.78
Metallic foam	3.7689e-11	66000	1.160e-11	0.658	55.47	2.78
Micro-machined disks	4.0000e-11	192000	2.873e-12	8.453	26.8	2.78

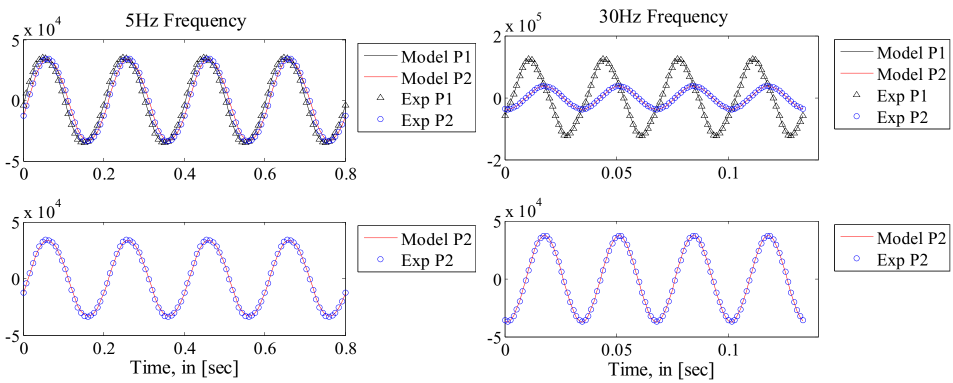

FIGURE 3. Prediction of pressure amplitude (P- P_{mean}), in [Pa] and phase at P_2, and their comparison to experimental data for metal foam, 5 Hz and 30 Hz.

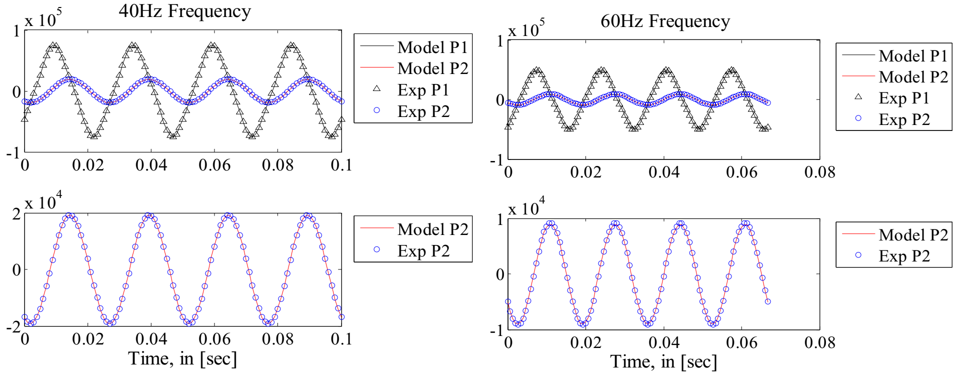

FIGURE 4. Prediction of pressure amplitude (P- P_{mean}), in [Pa] and phase at P_2, and their comparison to experimental data for metal foam, 40 Hz, and 60 Hz.

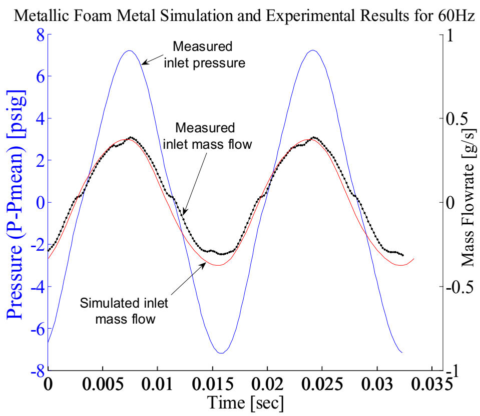

FIGURE 5. Measured inlet pressure and mass flow rate and predicted inlet mass flow rate for the metal foam filler for 60 Hz frequency.

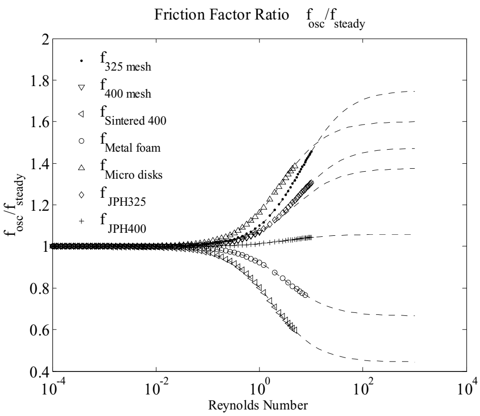

FIGURE 6. Longitudinal oscillatory to steady friction factor ratio. (Steady flow data were based on Clearman [9] and Harvey [10])

higher Reynolds numbers. For the 325 mesh screen regenerator, for example, the extrapolations of the data indicate that this ratio will reach approximately 1.7, confirming the importance of oscillatory friction factor.

CONCLUSION

We have established a systematic experimental and CFD-based procedure for the quantification of lateral permeability and Forchheimer's coefficients for various regenerator fillers under steady periodic flow conditions. In the investigation reported here, the methodology was applied for the quantification of the longitudinal flow parameters most widely used regenerator fillers. Periodic pressure drops were measured in an apparatus that consisted of a modular regenerator housing module containing regenerator, for inlet pressure oscillations with 5 to 60 Hz frequencies. By iterative CFD simulations, the aforementioned hydrodynamic parameters were calculated and correlated for several widely-used regenerator fillers. The results show that except for very low oscillation frequencies and the sintered type regenerators the cycle-average friction factor under oscillatory flow conditions is typically larger than the steady-flow friction factor.

ACKNOWLEDGEMENTS

This work was supported by the Georgia Tech and Raytheon Crycooler Research Foundation. We thank Drs. Jeremy P. Harvey and Prateen V. Desai for technical assistance. We also thank Dr. Kevin Kelley of International Mezzo Technology Inc. and Dr. Thomas Roberts of AFRL for providing the micro-machined disk regenerators.

NOTATION

$\overline{\overline{C}}$ = Inertial resistance coefficient tensor
f = Friction factor [-], frequency [Hz]
h = Enthalpy
$\overline{\overline{I}}$ = Unit identity tensor
k = Thermal conductivity
L = Regenerator length
p = Static pressure
T = Temperature
$\vec{u}$ = Intrinsic velocity

Greek letters
$\overline{\overline{\beta}}$ = Viscous resistance coefficient tensor
ε = Porosity Tensors
μ = Absolute viscosity
ρ = Density

Subscripts
f = Fluid
i =1 and 2
r, x = Radial coordinate, Axial coordinate
s = Solid

REFERENCES

1. Hozumi, Y., Shiraishi, M., and Murakami, M., "Simulation of thermodynamics aspects about pulse tube regenerators," in Advances in Cryogenic Engineering 49, edited J. Waynert et al., pp. 1500-1507.
2. Flakes, B., and Razani, A., "Modeling pulse tube cryocoolers with CFD," in Advances in Cryogenic Engineering 49, edited J. Waynert et al., pp. 1493-1499.
3. Cha, J. S., 2004. CFD Simulation of Multi-dimensional Effects in an Inertance Tube Pulse Tube Refrigeration, Masters Thesis, Georgia Institute of Technology, Atlanta, Ga.
4. Cha, J. S. 2007. Hydrodynamic Parameters of Micro-Porous Media for Steady and Oscillatory Flow: Application to Cryocooler Regenerators. PhD Thesis. Georgia Institute of Technology, Atlanta.
5. Jeong, S., Nam, K. and Jung, J. 2002. Regenerator Characterization under Oscillating Flow and Pulsating Pressure. Cryocoolers 12. pp. 531-537.
6. Nam K. and Jeong S. 2002. Experimental Study on the Regenerator under Actual Operating Conditions. Advances in Cryogenic Engineering. Vol. 47. pp. 977-984.
7. Roberts, T. and Desai P. 2003. Working Fluid State Properties Measurements in Medium and High Frequency Cryocoolers. Advances in Cryogenincs 49. pp.
8. Fluent User Manual version 6.2
9. Clearman, W.M. 2007. Measurement and Correlation of Directional Permeability and Forchheimer's Inertial Coefficients of Micro Porous Structures Used in Pulse Tube Cryocoolers. MS Thesis Georgia Institute of Technology, Atlanta Ga, 30332.
10. Harvey J.P. 2003. Oscillatory and Compressible flow and Heat Transfer in Porous Media- Application to Cryocooler Regenerators. PhD Thesis. Georgia Institute of Technology, Atlanta Ga, 30332.